**Colloids: phase behaviour and self-assembly – Problems Day 1**

**Exercise 1: Colloids in external fields**

Given that the barometric distribution of colloids as a function of height is described by the Boltzmann distribution

$$\rho = \rho_0 \ e^{-h/l_g}$$

with

$$l_g = \frac{kT}{m_b g}$$

Where $\rho$ is the particle density is the particle density at a height *h* above a reference level with $\rho_0$.Here $m_b = m - m'$ is the buoyant mass, with *m'* the mass of the fluid displaced by a particle of mass *m.*

a) Show that for colloidal particles dispersed in a liquid, the equilibrium number of particles, *N*, is given by:

$$N = N_0 \exp\,[-\frac{(m-m')g(h-h_0)}{k_b T}]$$

Where $N_0$ is the number of particles at height $h_0$.

b) In a tube of height 10 cm spherical colloids with a radius R = 10 nm are dispersed in water ($\rho_w$ = 1.0 g cm$^{-3}$). The particles have a density of $\rho_p$ = 1.2 g cm$^{-3}$. What is the ratio between the particle concentration at the top h = 10 cm and the bottom $h_0$ = 0 cm, after equilibrium has established. Assume the temperature is T = 20 °C.

c) Compute $l_g$ for particles with R = 90 nm using the same densities as above and assess whether the particles will settle in a sample cell of height 10 cm, 1 cm and 100 um.

d) Svedverg (1928) gives the following table for the sedimentation equilibrium of a gold sol under gravity.

| Height (um) | Number of particles | Height (um) | Number of particles |
|---|---|---|---|
| 0 | 889 | 600 | 217 |
| 100 | 692 | 700 | 185 |
| 200 | 572 | 800 | 152 |
| 300 | 426 | 900 | 125 |
| 400 | 357 | 1000 | 108 |
| 500 | 253 | 1100 | 78 |

Assume the particles have a radius *R* = 21 nm and density $\rho_p$ = 19.3 g cm$^{-3}$ and the temperature is T = 20 °C. Estimate the Boltzmann constant, $k_b$, from the equation derived in (a) and then calculate Avogadro's number, $N_A$, assuming R = 8.31 J K$^{-1}$ mol$^{-1}$.

e) Repeat the calculation with a radius of 22 nm and note how sensitive the answer is to this variable.