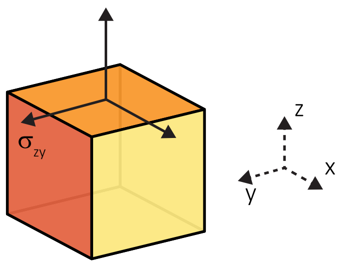
The problem set for this course consists of open questions for every lecture. The first lecture covers inertial numbers and the general context of granular materials. The second lecture covers experimental aspects, very slow flows and suspensions. Questions are organized accordingly.

Open Questions

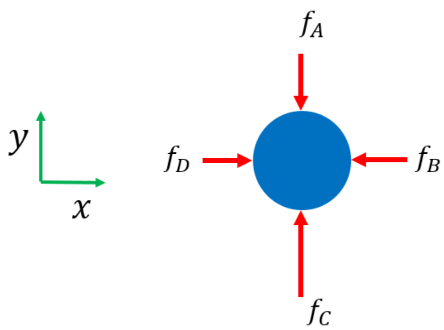
Background

1. 1. Which essential feature of granular materials has not been mentioned much in the lecture?

   2. Complete the following picture.



   3. Calculate the approximate total time it takes a particle with diameter d to traverse a distance d given that it is exposed to a pressure P and has a density ρ. This time scale is the microscopic time scale used in the definition of the inertial number I.



   4. The static blue disk-shaped particle shown above is under influence of four diametrically oriented contact forces f_i. If we know that the dyadic product $\otimes$ is defined as

$$\vec{a} \otimes \vec{b} = \begin{pmatrix} a_x b_x & a_x b_y \\ a_y b_x & a_y b_y \end{pmatrix}$$

   What is then the expression (in matrix form) for the stress tensor $\overline{\sigma}$ of the particle? Assume the disk radius is 1. Does this expression make sense?

Experimental Methods

2\.
1. Which factor (or factors) determines the minimum frame rate that you need during a granular flow experiment?
2. What is the difference between particle image velocimetry and particle tracking velocimetry? Look it up if you do not known what these terms mean.
3. What confines a material to the gap in a cone-plate or plate-plate experiment? Is this relevant for suspensions flows? How can you estimate its relevance?
4. What is a typical worry one should have when using a cone-plate setup with granular materials?
5. Describe how active materials fit into the list of discrete element material properties discussed in the lecture.
6. The lectures discussed rotating drums as experimental method to prove quasi-2D flow fields. Of course a drum is 3D. How many particle layers are needed to not find the effect of the wall? Try to find the answer to this question in the scientific literature (provide a reference or DOI).
7. It is possible to download your own Discrete Element Method code from the following site and try this out if computer simulations are what you like. See `http://guilhem.mollon.free.fr/Accueil_Eng.html`.

Non-local models

3. We want to know the flow field $\dot{\gamma}$ by solving the equation

$$\sigma(r) = \frac{G}{f(r)}\dot{\gamma}(r),$$

where we understand that $\sigma(r)$ is the local shear stress acting on a material element, f is a fluidity field taking the role of an inverse viscosity, and $\dot{\gamma}$ is the local shear rate in the material. We furthermore assume the standard formulation of the fluidity field, in which the deviations from "ordinary" flow behavior spread diffusively:

$$f(r) - f_{loc}(r) = \xi^2 \triangle f.$$

Here is the Laplacian and we assume that $f_{loc}(r)$ is set by the local rheology of a material, as set by microscopic interactions and as it would experience it in a small homogeneous environment:

$$f_{loc}(r) = \frac{\dot{\gamma}(\sigma)}{\sigma(r)}.$$

1. For this context, first write down the equation for $\dot{\gamma}$ for a Herschel-Bulkley fluid and its subcategory Bingham plastic.
2. What happens for $\sigma > \sigma_0$, so for local stresses that are above the yield stress?
3. Now constraining ourselves even more, to a cylindrical Couette system, which equation expresses the strength of the local stress $\sigma(r)$ in the gap? Write T for torque applied and H for the height of the cylinder. Feel free to look up the equation online.
4. To write down an equation for the fluidity, we need to know the Laplacian in cylindrical coordinates. Find it.
5. Combine the last two results to write down an explicit differential equation for f.
6. How is this equation now turned into an equation for the flow velocity $v(r)$?
7. Where is the yield stress in the gap?
8. is ξ constant?
9. For granular non-local models, one usually describes g, not f, the latter being used mostly in the context of emulsions. What is the difference between the two descriptions? item For a nice intro into the "why" of strain gradient models, you can check out:
   `https://doi.org/10.1007/s00419-002-0202-4`
   Especially the derivation of the strain gradient term for a periodic lattice is easy to follow and explains the emergence of a length scale, which for non-crystalline materials is of course more tricky to identify.

Wet granular materials

4. Granular materials are affected by the medium they are in.

    1. For example, viscous damping affects the time scale of the microscopic rearrangements. Derive the “viscous inertial” timescale assuming completely overdamped dynamics, meaning that the particle instantaneously achieves its terminal velocity while in the process of rearranging. Consider that for the particle to move, an opposite flow of the same volume/speed needs to occur, inside a porous medium. How large is the Darcy pressure of a fluid moving at a certain velocity through a porous medium?
    2. How does the Darcy pressure affect the motion of a single grain moving around in the granular medium?
    3. How does the porosity depend on particle size?
    4. what is then the typical microscopic time scale that we can use to write down a “viscous” Inertial number I_v?
    5. Calling this term a “viscous Inertial number is of course somewhat funny: why?
    6. What other regime can one imagine to exist in the context of wet granular media, considering only fluid drag, inertia, et cetera?